For steady state shock fronts the phase velocity c_p is the same for all parts of the wave and Eq. (13) can be integrated to give the usual momentum jump condition

$$P - P_o = \rho_o(U-u_o)(u-u_o)$$

where c_p has been replaced by the shock velocity with respect to material coordinates, $U-u_o$. Further, since P is a function of u only, we have:

$$c_u = \frac{du}{dP}\left(\frac{\partial P/\partial t}{du/dP}\right)\frac{\partial P}{\partial h} = c_p$$

Thus, Eq. (12) becomes

$$d\rho = [\rho^2/\rho_o(U-u_o)]du$$

Integrating yields the continuity jump condition

$$1 - \rho_o/\rho = (u-u_o)/(U-u_o)$$

The stress-volume curve is seen to be a straight line joining the end states (Rayleigh line) for, integrating Eq. (15) yields,

$$U - u_o = V_o\sqrt{(P-P_o)/(V_o-V)} = \text{const.}$$

Finally, Eq. (16) yields the Rankine-Hugoniot relation:

$$E - E_o = (1/2)(P+P_o)(V_o-V)$$

Note that this derivation does not require that the states be equilibrium states (except insofar as steady state implies equilibrium end states). Moreover, if the shock front is considered to be a discontinuity then $c_p = c_u$ and the jump conditions hold even if the flow behind the wave is unsteady.

C. Isentropic Flow in Fluids

The development of non-linear wave propagation theory in fluids relies heavily on the method of characteristics.[8] This method, in turn, depends on the assumption that the flow is everywhere particle-isentropic and that, therefore, all states are equilibrium states.

If to Eqs. (7) and (8) we add the relations:

$$(\partial S/\partial t)_h = 0 \tag{18}$$

and

$$P = P(\rho, S) \tag{19}$$

where S is entropy, we can write:

$$(\partial P/\partial \rho)_S = a^2 \quad \text{on } h = \text{const.},$$

where a is the sound speed with respect to spatial (Eulerian) coordinates. This relation allows us to eliminate the derivative in ρ from Eq. (7):

$$(\rho_o/\rho^2)(\partial\rho/\partial P)(\partial P/\partial t) + \partial u/\partial h = 0$$

or

$$\partial P/\partial t + (\rho^2 a^2/\rho_o)(\partial u/\partial h) = 0 \tag{20}$$

Multiplying Eq. (8) by a,

$$\rho a(\partial u/\partial t) + (\rho a/\rho_o)(\partial P/\partial h) = 0 \tag{21}$$

Adding and subtracting these gives

$$[(\partial/\partial t) \pm (\rho a/\rho_o)(\partial/\partial h)]P \pm \rho a[(\partial/\partial t) \pm (\rho a/\rho_o)(\partial/\partial h)]u = 0$$

The original equations are now reduced to a directional derivative and